Esercizi sul principio di induzione

Esercizio 1. *Dimostrare che* $1+2+3+...+n=\dfrac{n(n+1)}{2}\quad\forall$ *intero* $n\geq 1$.

Esercizio 2. *Dimostrare che* $1^2+2^2+3^2+4^2+...+n^2=\dfrac{n(n+1)(2\,n+1)}{6}\quad\forall$ *intero* $n\geq 1$.

Esercizio 3. *Dimostrare che* $1^3+2^3+3^3+4^3+...+n^3=\dfrac{n^2(n+1)^2}{4}\quad\forall$ *intero* $n\geq 1$.

Esercizio 4. *Dimostrare che* $1+3+5+7+...+(2\,n-1)=n^2\quad\forall$ *intero* $n\geq 1$.

Esercizio 5. *Dimostrare che* $1-x^n=(1-x)(1+x+x^2+...+x^{n-1})$ *con* $x\in\mathbb{R}\ \ \forall$ *intero* $n\geq 2$.

Esercizio 6. *Dimostrare che* n^3-n+6 *è divisibile per* $3\ \ \forall\, n\in\mathbb{N}$.

Esercizio 7. *Dimostrare che* $4^{2n+1}+3^{n+2}$ *è divisibile per* $13\ \ \forall\, n\in\mathbb{N}$.

Esercizio 8. *Dimostrare che, se* $x>-1$, $(1+x)^n\geq 1+nx\ \ \forall\, n\in\mathbb{N}$.

Esercizio 9. *Dimostrare che* $n!>2^n\quad\forall$ *intero* $n\geq 4$.

Esercizio 10. *Dimostrare che*

$$\left(1-\frac{1}{2}\right)\cdot\left(1-\frac{1}{3}\right)\cdot...\cdot\left(1-\frac{1}{n}\right)=\frac{1}{n}\quad\forall \text{ intero } n\geq 2\ .$$

Esercizio 11. *Dimostrare che*

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+...+\frac{1}{n\cdot(n+1)}=1-\frac{1}{n+1}\quad\forall \text{ intero } n\geq 1\ .$$

Esercizio 12. *Dimostrare che, se* $k\neq 1$, $(1+k)\cdot(1+k^2)\cdot(1+k^3)\cdot...\cdot(1+k^{2^n})=\dfrac{1-k^{2^{n+1}}}{1-k}\ \forall\, n\in\mathbb{N}$.

Esercizio 13. *Dimostrare che*

$$1\cdot 5^1+2\cdot 5^2+3\cdot 5^3+...+n\cdot 5^n=\frac{5+(4\,n-1)5^{n+1}}{16}\quad\forall \text{ intero } n\geq 1\ .$$

Esercizio 14. *Dimostrare che*

$$\frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\frac{1}{3\cdot 4\cdot 5}+...+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}\quad\forall \text{ intero } n\geq 1\ .$$

Esercizio 15. *Dimostrare che*

$$\left(1-\frac{1}{4}\right)\cdot\left(1-\frac{1}{9}\right)\cdot\left(1-\frac{1}{16}\right)\cdot...\cdot\left(1-\frac{1}{n^2}\right)=\frac{1+n}{2\,n}\quad\forall \text{ intero } n\geq 2\ .$$

Esercizio 16. *Dimostrare che*

$$1\cdot 2+2\cdot 3+3\cdot 4+...+n(n+1)=\frac{n(n+1)(n+2)}{3}\quad\forall \text{ intero } n\geq 1\ .$$

Esercizio 17. *Dimostrare che* $1\cdot 1!+2\cdot 2!+3\cdot 3!+...+n\cdot n!=(n+1)!-1\quad\forall$ *intero* $n\geq 1$.

Esercizio 18. *Dimostrare che*

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+...+\frac{1}{2\,n-1}-\frac{1}{2\,n}=\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+...+\frac{1}{2\,n}\quad\forall \text{ intero } n\geq 1\ .$$

Esercizio 19. *Dimostrare che* $\dfrac{1}{2}+\dfrac{2}{2^2}+\dfrac{3}{2^3}+...+\dfrac{n}{2^n}=2-\dfrac{n+2}{2^n}\quad\forall$ *intero* $n\geq 1$.